The Mark Ortiz Automotive

CHASSIS NEWSLETTER

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WELCOME

Mark Ortiz Automotive is a chassis consulting service primarily serving oval track and road racers. This newsletter is a free service intended to benefit racers and enthusiasts by offering useful insights into chassis engineering and answers to questions. Readers may mail questions to: 155 Wankel Dr., Kannapolis, NC 28083-8200; submit questions by phone at 704-933-8876; or submit questions by e-mail to: markortizauto@windstream.net. Readers are invited to subscribe to this newsletter by e-mail. Just e-mail me and request to be added to the list.

BANKED TURN PUZZLE

I was just reading your "Measuring Grip" when it hit me that you might be able to help answer a question with respect to calculating the maximum speed of a car in a turn of a given radius, coefficient of friction and banking. The attached Exel doc contains a formula that I found on the internet that is supposed to be useful, but the speed goes infinite at 45 degrees of bank, given a coef of friction of 1, and I maintain that is wrong. My friend who has a masters degree in engineering from GA tech, and another degreed engineer, maintain that the equation is correct. (The equation contains cos theta - mu x sine theta in the denominator, mu = coef of friction)

Can you help?

The equation in question is:

$$v = \sqrt{\left(rg\,(\sin\theta + \mu\cos\theta)/(\cos\theta - \mu\sin\theta)\right)}$$

where:

v = maximum possible linear velocity of the car, ft/sec or M/sec
r = radius of turn, gravitationally horizontal, ft or M
g = acceleration of gravity, ft/sec^2 or M/sec^2
θ = angle of banking, from gravitational horizontal
μ = coefficient of friction

It will be apparent that for $\mu = 1$, the denominator goes to zero when $\theta = 45^\circ$, and v becomes undefined.

It is counterintuitive that the car should have no limiting speed if the banking isn't vertical. It doesn't look right, but it is right. However, for a 45° banking, it's only true if μ is at least 1.

At 45°, what happens is that at 1g horizontal centripetal acceleration (a_H = 1g), no cornering force is required of the tires. When speed drops below the value corresponding to 1g horizontal, the car tries to slide down the banking and the tires must exert a negative cornering force. As the speed rises above the 1g value, the tires must exert a positive cornering force, and you'd think at some speed their grip limit will be exceeded.

But let's try some numbers, not using the spreadsheet or the equation, but just using trigonometry and our own brains.

$\mu = 1$ is the minimum requirement for the car not to slide down the banking at a standstill.

At $a_H = 0$, the normal force on the tires is $W/(\sqrt{2})$, force down the banking due to gravity is $W/(\sqrt{2})$, and force up the banking is 0.

At a_H = 1g, the normal force on the tires is $W/(\sqrt{2}) + W/(\sqrt{2})$, or $(\sqrt{2})W$, force down the banking due to gravity is $W/(\sqrt{2})$, and force up the banking is $W/(\sqrt{2})$.

At a_H = 2g, the normal force on the tires is $W/(\sqrt{2}) + (\sqrt{2})W$, or $(3/\sqrt{2})W$, force down the banking due to gravity is $W/(\sqrt{2})$, and force up the banking is $(\sqrt{2})W$ for a required cornering force of $W/(\sqrt{2})$. The car can do that.

At a_H = 4g, the normal force on the tires is $W/(\sqrt{2}) + (2\sqrt{2})W$, or $(5/\sqrt{2})W$, force down the banking is still $W/(\sqrt{2})$, and force up the banking is $(2\sqrt{2})W$. The car can do that.

Now we can see the pattern that's emerging. The normal force is always greater than the induced load due to banking by $W/(\sqrt{2})$, and the net force up the banking is always equal to the induced load *minus* $W/(\sqrt{2})$. So although the ratio between the cornering force needed and the normal force asymptotically approaches 1, it never gets there. So there will be a limiting speed at some point on a 45° banking if μ is less than 1, but not if it's greater than or equal to 1.

The banking angle where there is no upper limiting speed for any μ is vertical. However, there will then be a minimum speed to keep from sliding down. As μ diminishes, that minimum speed approaches infinity.

In fact, I think we can say that when $\mu = 1$, there is exactly one banking angle with no minimum or maximum speed. When $\mu < 1$ or $\mu = 1$, any banking angle less than 45° has a maximum speed, and any angle over 45° has a minimum speed. When $\mu > 1$, there will be a band of banking angles around 45° with no maximum or minimum.

All of this of course ignores certain realities of tire behavior. μ isn't constant, and we can't just go on adding normal force without failing the tires.

It also ignores the realities of paving such a banking. No race track in the world has such a banking. Talladega is supposed to be 31 degrees. Daytona is supposed to be 30. Charlotte is 23. I have been on the Charlotte banking. It feels like walking on a roof. The track gives van ride-arounds to tourists. As part of the ride, the van stops on the banking. It feels like you're going to tip over, into the infield.

If you wanted to pave a banking at 45˚, how would you keep the paving machine from sliding down the banking? How would you even grade such a banking?

At 23˚, it is difficult to keep asphalt smooth. When the sun shines on the banking, the asphalt softens and starts to sag down the banking due to its own weight. That's why tracks that are any steeper are generally concrete.

In other words, the problem posed here is theoretical. In the world we actually live and race in, you can't just go faster without limit, on any existing track.

Still, the theoretical physics of such a hypothetical case are interesting.

So are the civil engineering aspects of trying to build a track that steep. No doubt it can be done, and even has been done, after a fashion.

I have seen very small, very steep, bowl-shaped circular board tracks at fairs, where motorcycles are ridden around at giddy angles. These are actually portable facilities. They can be taken apart and moved as the midway show travels.

Board tracks for full-sized cars enjoyed brief popularity in the 1920's in the US. Some of these were very steep. They didn't age gracefully. After a few years, boards started coming loose and impaling people, and board tracks were quickly abandoned.

However, if somebody wanted to build a really steeply banked track today, perhaps the board track concept has some ideas to offer.

I don't mean the idea of using wood as the material, but perhaps the idea of assembling the track surface out of pieces formed off-site, or at least not formed in place. It might be possible to make the surface as pre-cast sections of reinforced concrete deck, and then move them into place with a crane. These segments could rest on an earthen roadbed, or a further idea could be borrowed from the board tracks and the sections could be supported on an above-ground framework. The framework could incorporate adjustment to position the segments as things settled, and smooth out the track surface. Sections could be replaced individually. Cracks between sections could be left unsealed for drainage.

Rather like the physics of driving on very steep bankings, this is fun to think about, but I don't plan on holding my breath until somebody does it.

REVERSE-CANT REAR LEAF SPRINGS

What would the effect be if the leaf springs in a road racing vehicle were to have the cant or angle of the leaf springs reversed? ... i.e. The springs are normally closer together at the front pivot point than the rear pivot ....It is my understanding that that the leaf from the front pivot to axle simply acts as a control arm?

With respect to torque reaction, the action of the spring is more complex than just the front half acting as a control arm. In top view, as regards deflection steer with lateral force, the system does tend to mainly yield laterally at the rear portion of the spring and the shackle. As a crude approximation, the axle pivots around an instant center found by projecting the top view centerlines of the springs.

If, as is common, that instant center is ahead of the axle, the system tends to have deflection oversteer: the deflection points the rear wheels out of the turn. If the springs cant out at the front, we get deflection understeer instead.

This could be a good thing. In general, it's better to have deflection understeer in a car than deflection oversteer.

One often sees the view expressed that canting the springs increases the lateral rigidity of the system. I think that would be true if some magic force kept the axle from steering with lateral deflection. But since there is nothing preventing the axle from steering except the springs, within any practical angles, with the springs canted the system still deflects, and we get some deflection steer when it happens.